Question 1

$$f(x) = 2x^2 - 8x + 14, \ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are integer constants. **(4)**

b) Find the coordinates of the minimum point on the curve with equation …

i. … $y = f\left(\frac{1}{2}x\right)$. **(2)**

ii. … $y = f(x+1) - 4$. **(2)**

Question 5

Solve the following system of simultaneous equations

$$\left(x + y\sqrt{3}\right)^2 = 56 + 12\sqrt{3}$$

$$y = 3x.$$ **(6)**

Question 8

The points A and B have coordinates $(0,-4)$ and $(3,-2)$, respectively.

a) Determine an equation for the straight line l which passes through the points A and B, giving the answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The point C lies on l, so that the distance AC is $3\sqrt{13}$ units.

b) Show, by a complete algebraic solution, that one possible set of coordinates for C are $(9,2)$ and find the other set. **(10)**

Question 11

$$f(x) = 4x^2 + 12kx,\ x \in \mathbb{R},$$

where k is a constant.

a) Show clearly that the equation $f(x) = 9$ has two distinct real roots for all values of k. **(3)**

b) Hence find the solutions of the equation $f(x) = 9$, giving the answers in the form $pk \pm p\sqrt{k^2+1}$, where p is a constant to be found. **(3)**

SOLUTIONS

1. a) SIGHT OF x^2-4x+7 M1

$a=2 \quad b=-2 \quad c=6$ B3

b I) $(4,6)$ A1 A1

II) $(1,2)$ A1 A1

5. $(x+3x\sqrt{3})^2 = 56+12\sqrt{3}$ M1

$x^2(1+3\sqrt{3})^2$ OR $x^2+6\sqrt{3}x^2+27x^2$ M1

SIGHT OF $28+6\sqrt{3}$ B1

$\frac{56+12\sqrt{3}}{28+6\sqrt{3}} = 2$ A1

$x = \pm\sqrt{2}$ A1

$y = \pm 3\sqrt{2}$ A1

8. a) $\frac{-2-(-4)}{3-0}$ o.e M1

$\frac{2}{3}$ A1

$y=\frac{2}{3}x-4$ OR $2x-3y-12=0$ A1

b) $\left(x, \frac{2}{3}x-4\right)$ MUST BE AS CO-ORDINATES B1

$\sqrt{\left[-4-\left(\frac{2}{3}x-4\right)\right]^2+(0-x)^2}\left(=3\sqrt{13}\right)$ M1 USE OF FORMULA
M1 ALL CORRECT

$\sqrt{\frac{4}{9}x^2+x^2}\left(=3\sqrt{13}\right)$ A1

$\sqrt{\frac{13}{9}x^2}\left(=3\sqrt{13}\right)$ A1

$\frac{13}{9}x^2=117$ OR $\frac{13}{9}x^2=9\times 13$ M1

$x^2=81$ M1

$x=\pm 9$ A1

$(9,2)$ $(-9,-10)$ A1 A1

WITH EVIDENCE OF SUBSTITUTION

11. a) $(12k)^2-4\times 4\times(-9)$ M1

$144k^2+144$ A1

IMPLIES THIS IS ALWAYS POSITIVE / OR AT LEAST 144 SO A1

b) $\frac{-12k\pm\sqrt{144k^2+144}}{2\times 4}$ o.E M1 (ALLOW SIMILAR IF COMPLETING THE SQUARE)

SIGHT OF $12\sqrt{k^2+1}$ B1

$-\frac{3}{3}\pm\frac{3}{2}\sqrt{k^2+1}$ A1 c.a.o